

FYS3610

EXERCISES WEEK 40

Midterm exam from autumn 2006. You should be able to answer all questions except problem 2e.

DEPARTMENT OF PHYSICS, UIO

FYS3610-SPACE PHYSICS

MID-TERM EXAMINATION

Date: October 9, 2006

Time of day: 13:30-15:30 (2 hours!)

Permitted aid(s): Calculating machine.

The set consists of 4 pages, with 3 Problems.

NOTE: At page 4 you find a Table containing useful information.

PROBLEM 1

- a) Sketch a typical electron density (m^{-3}) versus altitude profile for daytime sunlit conditions. Mark out the altitude ranges for the D-, E-, and F-layers.

The ion and electron momentum equations are given as:

$$n_i m_i \frac{d\vec{v}_i}{dt} = n_i e (\vec{E} + \vec{v}_i \times \vec{B}) - n_i m_i \nu_{in} \vec{v}_i \quad (1.1)$$

$$n_e m_e \frac{d\vec{v}_e}{dt} = -n_e e (\vec{E} + \vec{v}_e \times \vec{B}) - n_e m_e \nu_{en} \vec{v}_e \quad (1.2)$$

- b) Describe the different terms in Eq. 1.1. Explain briefly how the following expressions for \vec{v}_e and \vec{v}_i can be derived:

$$\vec{v}_i = \frac{\omega_i v_{in}}{\omega_i^2 + v_{in}^2} \vec{E}_\perp + \frac{\omega_i^2}{\omega_i^2 + v_{in}^2} \frac{\vec{E}_\perp \times \vec{B}}{B^2} \quad (1.3)$$

$$\vec{v}_e = -\frac{\omega_e v_{en}}{\omega_e^2 + v_{en}^2} \vec{E}_\perp + \frac{\omega_e^2}{\omega_e^2 + v_{en}^2} \frac{\vec{E}_\perp \times \vec{B}}{B^2} \quad (1.4)$$

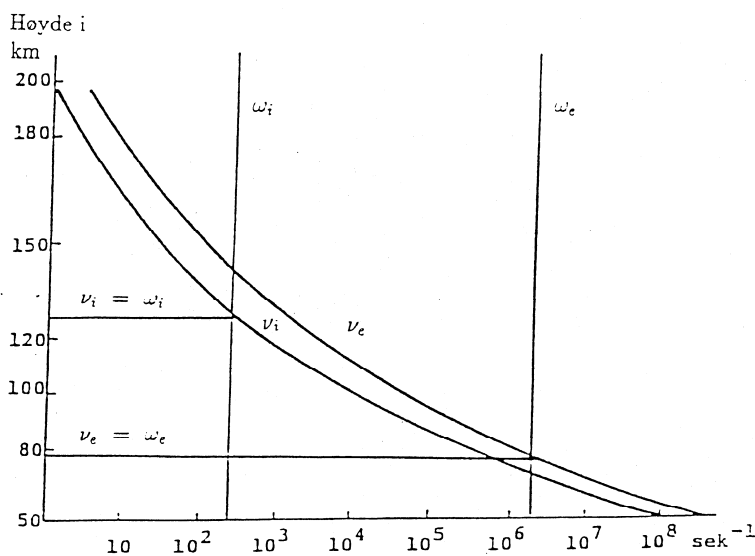


Figure 1.1

- c) Explain Figure 1.1 above.
- d) Apply Figure 1.1, Eq. 1.3 and Eq. 1.4 to describe rotation of the velocity vectors \vec{v}_i and \vec{v}_e by altitude. (Hint: Derive expressions for the angle between the velocity vectors and \vec{E}_\perp and draw an illustration). Derive expressions for the electron and the ion speeds and comment on the altitude variation. What is the maximal speed?
- e) Find an expression for the current density \vec{j} . Point out the Hall and Pedersen terms and discuss \vec{j} versus altitude. What controls the upper and the lower limits of the conductive layer?

PROBLEM 2

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad (2.1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2.2)$$

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \quad (2.3)$$

- What are the well-known names of Eqs. 2.1-2.3. Define the parameters involved.
- Derive an expression for $\frac{\partial \vec{B}}{\partial t}$ and show that the magnetic Reynold's number is given by $R_m = \mu_0 \sigma vL$.
- Discuss the physical implications of the $R_m \ll 1$ and $R_m \gg 1$.
- Make a brief discussion of the frozen-in-field concept. Give an example where the frozen-in-field concept breaks down.
- Give a cartoon description of a CME event.

PROBLEM 3

Suppose that the earth's magnetic field is 3×10^{-5} T at the equator and falls off as $1/r^{-3}$ as for a perfect dipole. Let there be an isotropic population of 1-eV protons and 30-keV electrons, each with a density of $n = 10^7 \text{ m}^{-3}$ at $r = 5$ earth radii in the equatorial plane. The general expression for the gradient drift is given as:

$$\vec{u}_{\nabla B} = \frac{1}{2} m v_{\perp}^2 \frac{\vec{B} \times \nabla B}{qB^3} \quad \text{Eq. 3.1}$$

- Compute the ion and electron gradient drift velocities.
- Does the electron drift eastward or westward?
- How long does it take for the electron to encircle the earth?
- Compute the ring current density Am^{-2} .

Table:

$$E = \frac{1}{2}mv_{\perp}^2$$

$$1\text{eV} = 1.6 \times 10^{-19} \text{ J}$$

$$B_{\text{Eq}} = B_0 \left(\frac{R_E}{r} \right)^3$$

$$B_0 = 30000 \text{ nT}$$

$$R_E = 6400 \text{ km}$$

$$m_p = 1.67 \times 10^{-27} \text{ kg}$$

$$m_e = 9.1 \times 10^{-31} \text{ kg}$$

$$c = 3 \times 10^8 \text{ ms}^{-1}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$$